Worksheet for Week 1 - Precalc

Instructions. DISCUSS with your group mates, and do the problems below. You are not expected to finish all the problems. So take your time. :)

1. Some of these algebraic manipulations are correct, and others are incorrect. (Note that a correct answer might be obtained by using incorrect reasoning.) Identify the correct ones and fix the incorrect ones:

F (a) $\dfrac{(x+y)^2}{y} \neq \dfrac{x^2+y^2}{y} \neq x^2+y.$ $(x+y)^2 = x^2+2xy+y^2$

T (b) $\dfrac{5}{\sqrt{x+5}-\sqrt{x}} = \dfrac{5\left(\sqrt{x+5}+\sqrt{x}\right)}{\left(\sqrt{x+5}-\sqrt{x}\right)\left(\sqrt{x+5}+\sqrt{x}\right)} = \dfrac{5\left(\sqrt{x+5}+\sqrt{x}\right)}{5} = \sqrt{x+5}+\sqrt{x}.$ $(a-b)(a+b)=a^2-b^2$

F (c) $\dfrac{x^2-1}{x+1} \neq \dfrac{x^2}{x} - \dfrac{1}{1} = x-1.$

F (d) $\dfrac{x^{-1}+y^{-1}}{x^{-1}-y^{-1}} \neq \dfrac{(x+y)^{-1}}{(x-y)^{-1}} = \left(\dfrac{x+y}{x-y}\right)^{-1} \neq -\dfrac{x+y}{x-y}.$

F (e) $(-3)^2 = \neq 3^2 = -9.$

F (f) $2^a 2^{-b} = 2^{-ab}.$ $x^a x^b = x^{a+b}$

T (g) $\ln a + \ln b = \ln(ab)$, $\ln a - \ln b = \ln\left(\frac{a}{b}\right).$

2. Now it's the time to revisit some knowledge of exponential and logarithms. One example:

$2^3 = 8$ is equivalent to say $\log_2 8 = 3$.

What are the following expressions?

$\log_3 27 =$ 3 $\quad 3^3 = 27$

$\log_4 16 =$ 2 $\quad 4^2 = 16$

$\log_{10} 1000 =$ 3 $\quad 10^3 = 1000$

$\log_{10} 0.001 =$ -3 $\quad 10^{-3} = \frac{1}{10^3} = 0.001$

$\log_7 1 =$ 0 $\quad 7^0 = 1$

$\log_{2017} 1 =$ 0 $\quad 2017^0 = 1$

Properties of logarithm

$\log_a(uv) = \log_a u + \log_a v$

$\log_a \frac{u}{v} = \log_a u - \log_a v$

$\log_a u^v = v \log_a u$

$f(x) = \ln x$ the domain is $x > 0$.

3. Still remember quadratic formula?

$ax^2 + bx + c = 0$. the solution is $x =$ $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

Solve for x such that

$$x + 1 = x(x - 2)$$

$x+1 = x^2-2x$

$x^2-3x-1=0$

$x = \frac{3 \pm \sqrt{9+4}}{2}$

$= \frac{3 \pm \sqrt{13}}{2}$

Notice: Although the quadratic equation ends up the same, this problem is different from #7 in webwork. Because one needs to consider the domain of "ln" in that problem.

4. *(homework question)* If $\ln a = 2, \ln b = 3$ and $\ln c = 5$ evaluate the following:

(a) $\ln\left(\frac{a}{b^2c^3}\right)$

(b) $\ln\sqrt{b^1c^{-3}a^{-2}}$

(c) $\frac{\ln\left(a^3b^{-3}\right)}{\ln\left((bc)^2\right)}$

(d) $(\ln\ c^3)\left(\ln\frac{a}{b^3}\right)^{-2}$

(a) $? = \ln a - \ln(b^2c^3)$

$= \ln a - \ln(b^2) - \ln(c^3)$

$= \ln a - 2\ln b - 3\ln c$

$= 2 - 2\times3 - 3\times5$

$= -19$

(b) $? = \frac{1}{2}\ln(bc^{-3}a^{-2})$

$= \frac{1}{2}[\ln b - 3\ln c - 2\ln a]$

$= \frac{1}{2}[3 - 15 - 4]$

$= -8$

(c) $= \frac{3\ln a - 3\ln b}{2(\ln b + \ln c)}$

$= \frac{6-9}{16}$

$= -\frac{3}{16}$

(d) $? = 3\ln c \cdot (\ln a - 3\ln b)^{-2}$

$= 15 \times (2-9)^{-2}$

$= 15 \times \frac{1}{(-7)^2}$

$= \frac{15}{49}$